

$$e^{At}$$

$$\frac{dx_1}{dt} = x_1 + 2x_2$$

$$\frac{dx_2}{dt} = 3x_1 + 4x_2$$

$$\bar{X} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad \frac{d\bar{X}}{dt} = \begin{bmatrix} \frac{dx_1}{dt} \\ \frac{dx_2}{dt} \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$$\det(A - \lambda I) = 0 \quad \begin{vmatrix} 1-\lambda & 2 \\ 3 & 4-\lambda \end{vmatrix} = 0$$

$$(1-\lambda)(4-\lambda) - (3)(2) = 0$$

$$\lambda^2 - 5\lambda + 4 - 6 = 0$$

$$\lambda^2 - 5\lambda - 2 = 0$$

$$\lambda = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(-2)(1)}}{2(1)}$$

$$\lambda = \frac{5 \pm \sqrt{25+8}}{2}$$

$$\lambda = \frac{5 \pm \sqrt{33}}{2}$$

$$\lambda_1 = \frac{5 + \sqrt{33}}{2}$$

$$\lambda_2 = \frac{5 - \sqrt{33}}{2}$$

$$e^{At} = B_0(t)I + B_1(t)A$$

$$e^{\lambda t} = B_0(t) + B_1(t)\lambda$$

$$e^{\left(\frac{5+\sqrt{33}}{2}\right)t} = B_0(t) + \left(\frac{5+\sqrt{33}}{2}\right)B_1(t)$$

$$e^{\left(\frac{5-\sqrt{33}}{2}\right)t} = B_0(t) + \left(\frac{5-\sqrt{33}}{2}\right)B_1(t)$$

$$e^{\left(\frac{5+\sqrt{33}}{2}\right)t} - e^{\left(\frac{5-\sqrt{33}}{2}\right)t} = \left[\left(\frac{5+\sqrt{33}}{2}\right) - \left(\frac{5-\sqrt{33}}{2}\right) \right] B_1(t)$$

$$B_1(t) = \frac{e^{\left(\frac{5+\sqrt{33}}{2}\right)t} - e^{\left(\frac{5-\sqrt{33}}{2}\right)t}}{\left(\left(\frac{5+\sqrt{33}}{2}\right) - \left(\frac{5-\sqrt{33}}{2}\right)\right)}$$

$$B_0(t) = e^{\left(\frac{5+\sqrt{33}}{2}\right)t} - \left(\frac{5+\sqrt{33}}{2}\right) \left[\frac{e^{\left(\frac{5+\sqrt{33}}{2}\right)t} - e^{\left(\frac{5-\sqrt{33}}{2}\right)t}}{\left(\left(\frac{5+\sqrt{33}}{2}\right) - \left(\frac{5-\sqrt{33}}{2}\right)\right)} \right]$$

$$\frac{dx(t)}{dt} = a_{11}x(t) + a_{12}y(t) + b_1(t)$$

$$\frac{dy(t)}{dt} = a_{21}x(t) + a_{22}y(t) + b_2(t)$$

$$S(2) \in \mathcal{D} \cap L(1) \subset \mathcal{N}H.$$

$$\bar{X}(t) = \begin{bmatrix} x(t) \\ y(t) \end{bmatrix} \quad \frac{d}{dt} \bar{X}(t) = \begin{bmatrix} \frac{dx(t)}{dt} \\ \frac{dy(t)}{dt} \end{bmatrix}$$

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \quad \bar{B}(t) = \begin{bmatrix} b_1(t) \\ b_2(t) \end{bmatrix} \quad \bar{X}(0) = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$$

$$\frac{d}{dt} \bar{X}(t) = A \cdot \bar{X}(t) + \bar{B}(t)$$

$$\bar{X}(t) = e^{A(t)} \bar{X}(0) + \int_0^t e^{A(t-\tau)} \bar{B}(\tau) d\tau.$$

$$\frac{dx(t)}{dt} = 4x(t) - 5y(t) + e^{2t} + t$$

$$\frac{dy(t)}{dt} = -6x(t) + 7y(t) + \cos(2t)$$

$$x(0) = 8 \quad y(0) = -4$$